



Course Title: Fundamentals of Stochastic Processes أسس العمليات العشوائية Course Code: CCER١١٧ ٣rd year
Date: ١٥.١.٢٠١٢ (First term) Allowed time: ٢ hrs No. of Pages: (٢)

Answer the following four questions. You are allowed to use the accompanying two tables of standard normal curve ordinates and areas in your answers.

Question No. ١

(١٦ marks)

- (a) Let $S=\{a, b, c, d, e, f\}$ with $P(a)=1/16$, $P(b)=1/16$, $P(c)=1/8$, $P(d)=2/16$, $P(e)=1/4$ and $P(f)=0/16$. Let $A=\{a, c, e\}$, $B=\{c, d, e, f\}$ and $C=\{b, c, f\}$. Find:
- $P(A/B)$.
 - $P(B/C)$.
 - $P(C/A^c)$.
 - $P(A^c/C)$.
- (b) Let A , B , and C be events. Find an expression, and exhibit the Venn diagram, for the event that:
- A and B , but not C occurs.
 - Only A occurs.
- (c) In a certain college, ٢٥% of the boys and ١٠% of the girls are studying mathematics. The girls constitute ٦٠% of the students. If a student is selected at random and is studying mathematics, determine the probability that the student is a girl?

Question No. ٢

(١٨ marks)

- (a) Find the expectation, variance, and standard deviation of the random variable x with density function $P(x)$ given as:

x	١	٢	٤	٥
$P(x)$	٠.٤	٠.١	٠.٢	٠.٣

- (b) Prove that for any random variable x :

- $E(ax + b) = a E(x) + b$
- $V(ax + b) = a^2 V(x)$
- $E(c) = c$
- $V(c) = 0$

where a , b , and c are constants.

- (c) If the density function $f(x)$ is given by:

$$f(x) = \begin{cases} 1-x & 0 \leq x \leq 1 \\ x-1 & 1 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

find the distribution function $F(x)$.

Question No. 7

(14 marks)

(a) A coin, weighted with $P(H) = \frac{2}{3}$ and $P(T) = \frac{1}{3}$, is tossed three times. Let x be a random variable denoting the longest string of heads that occurs. Find the distribution, expectation, variance, and standard deviation of x .

(b) Consider the following binomial probability distribution:

$$P(x) = \binom{5}{x} (0.7)^x (0.3)^{5-x} \quad (x = 0, 1, \dots, 5)$$

where x is a random variable.

- How many trials (n) are in the experiment?
- What is the value of p , the probability of success?
- Graph $P(x)$.
- Find the mean and standard deviation of x .

(c) Suppose 2% of items made by a factory are defective. Find the probability that there are 2 defective items in a sample of 100 items.

Question No. 8

(14 marks)

(a) Let x be a random variable with a standard normal distribution Φ . Find:

- $P(x \geq 1.12)$
- $P(0 \leq x \leq 1.24)$
- $P(0.60 \leq x \leq 1.26)$
- $P(-0.72 \leq x \leq 0)$

(b) Let x be a random variable with the standard normal distribution Φ . Determine the value of t , standard units, if:

- $P(0 \leq x \leq t) = 0.4236$
- $P(x \leq t) = 0.7967$
- $P(t \leq x \leq 1) = 0.1000$

(c) A class has 12 boys and 8 girls. If three students are selected at random one after the other from the class, what is the probability that they are all boys?

Best wishes

Midterm exam

Question 1

(1) Let A and B be events. Find an expression and exhibit Venn-diagram for the event that:

- (i) A but not B occurs i.e. only A occurs.
- (ii) Either A or B, but not both occurs.
- (iii) A or not B occurs
- (iv) Neither A nor B occurs

(2) Let a die be weighted so that the probability of a number appearing when the die is tossed is proportional to the given number let:

$$A = \{\text{even no.}\} \quad B = \{\text{prime no.}\} \quad C = \{\text{odd no.}\}$$

- (i) Find the probability of each sample point of the sample space
- (ii) Find $P(A)$, $P(B)$ and $P(C)$
- (iii) Find the probability that
 - (a) An even or prime number occurs
 - (b) An odd prime number occurs
 - (c) A but not B occurs

(3) Let A and B be events with $P(A) = 1/3$, $P(B) = 1/4$, and $P(A \cup B) = 1/2$

- Find: (i) $P(A|B)$ (ii) $P(B|A)$
(iii) $P(A \cap B^c)$ (iv) $P(A|B^c)$

(4) If the density function $f(x)$ is given by:

$$f(x) = \begin{cases} 1-x & 0 \leq x \leq 1 \\ x-1 & 1 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find the distribution function, and sketch both the distribution and the density functions.

Question 2

(1) The probability that a man will live 10 more years is $1/4$, and the probability that his wife will live 10 more years is $1/3$. Find the probability that:

- (i) both will live 10 more years
- (ii) at least one will live 10 more years
- (iii) neither will live 10 more years
- (iv) only the wife will live 10 more years

(2) Let X be a continuous random variable with the distribution

$$f(x) = \begin{cases} kx & \text{if } 0 \leq x \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

- (i) Evaluate k (ii) Find $P(1 \leq x \leq 3)$, $P(2 \leq x \leq 4)$, and $P(x \leq 3)$

Good Luck

Prof. Dr. E. Sallam

Answer all the following questions;

Question No. 1

(a) If A and B are independent events, prove that A^c and B are independent.

(b) Let A and B be events with $P(A) = 1/2$, $P(B) = 1/3$ and $P(A \cap B) = 1/4$.

Find : i- $P(A|B)$, ii- $P(B|A)$, iii- $P(A \cup B)$, iv- $P(A^c|B^c)$, v- $P(B^c|A^c)$

(c) If X be a continuous random variable with the probability

$$P(x) = x/2 \quad 0 < x < 2, \text{ and zero elsewhere}$$

Find the cumulative distribution function, mean, variance, and standard deviation of X.

(d) Given a and b are constants, find with prove i - $E(a) = ?$ ii - $\text{Var}(a + b) = ?$

Question No. 2

(a) Three light bulbs are chosen at random from 15 bulbs of which 5 are defective.

Find the probability that : i- exactly one is defective, ii- none is defective,
iii- at least one is defective iv- at most one is defective.

(b) Let X be a continuous random variable with distribution

$$f(x) = x/6 + k \quad \text{if } 0 \leq x \leq 3 \text{ and } f(x) \text{ equals zero elsewhere.}$$

Sketch the graph of $f(x)$ and thus i- Evaluate k ii- Find $P(1 \leq X \leq 2)$

(c) A pair of fair dice is tossed. Let X assigns to the sum of dices numbers.

Calculate the mean, variance and standard deviation of X.

(d) Let X be a random variable with the binomial distribution $b(k;n,p)$.

Prove that $E(X) = np$.

The Fundamentals of Stochastic processes

Sheet no.5

1) Medical research has shown that a certain type of chemotherapy is successful 70% of the time when used to treat skin cancer. suppose five cancer patients are treated with this type of chemotherapy and let x equal the no. of successful cures out of the five.

x	0	1	2	3	4	5
$P(x)$	0.002	0.029	0.132	0.309	0.360	0.168

The probability distribution of x is given in the following table.

Find:

a) $\mu = E(x)$

b) $\sigma = \sqrt{E(x - \mu)^2}$

2) Find the expectation, variance and the standard deviation of each of the following:

i)

x	2	3	11
$P(x)$	$1/3$	$1/2$	$1/6$

ii)

x	-5	-4	1	2
$P(x)$	$1/4$	$1/8$	$1/2$	$1/8$

iii)

x	1	3	4	5
$P(x)$	0.4	0.1	0.2	0.3

(b) A coin weighted so that $P(H) = 1/3$ and $P(T) = 2/3$ is tossed until a head or four tails occur.

Find the expected number of tosses of the coin.

(c) Determine the expected number of boys in a family with 8 children, assuming the sex distribution to be equally probable. What is the probability that the expected number of boys does occur?

(d) Let X be a random variable with the binomial distribution $b(k; n, p)$.

Prove that $E(X) = np$.

Question No. 4

(18 marks)

(a) Determine the expected number of boys in a family with 8 children, assuming the sex distribution to be equally probable. What is the probability that the expected number of boys does occur?

(b) Suppose the diameters of bolts manufactured by a company are normally distributed with mean 0.25 inches and standard deviation 0.02 inches. A bolt is considered defective if its diameter is ≤ 0.20 inches or ≥ 0.28 inches. Find the percentage of defective bolts manufactured by the company. 93.49%

(b) Suppose the heights of 1000 male students are normally distributed with mean 175 centimeters and standard deviation 20 centimeters.

Find the number of students with heights:

i- less than or equal to 130 centimeters, 12

ii- between 150 and 160 centimeters, 121

iii- between 170 and 180 centimeters 107

iv- greater than or equal to 200 centimeters. 106

Best wishes

Dr. Eng. Alsayed Sallam



Course Title: Stochastic Processes العمليات العشوائية ثلاثية حاسبات
Date: 4.2.2010 (First term)

Course Code: CCE3117 3rd year
Allowed time: 1 hrs No. of Pages: (2)

Answer all the following questions:

Question No. 1

(17 marks)

(a) If A and B are independent events, prove that A and B^c are independent.

(b) Let A and B be events with $P(A) = 1/3$, $P(B) = 1/2$ and $P(A \cap B) = 1/4$.

Find : i- $P(A|B)$, ii- $P(B|A)$, iii- $P(A \cup B)$, iv- $P(A^c|B^c)$, v- $P(B^c|A^c)$

(c) If X be a continuous random variable with the probability

$$P(x) = x/4 \quad 0 < x < 4, \text{ and zero elsewhere}$$

Find the cumulative distribution function, mean, variance, and standard deviation

(d) Given a and b are constants, find with prove i - $E(a) = ?$ ii - $\text{Var}(aX + b) = ?$

where X is a continuous random variable.

$$a^2 \text{Var}(X)$$

Question No. 2

(17 marks)

(a) Three light bulbs are chosen at random from 20 bulbs of which 5 are defective. Find the probability that : i- exactly one is defective, ii- none is defective,

iii- at least one is defective iv- at most one is defective. ✓

(b) Let X be a continuous random variable with distribution

$$f(x) = x/4 + k \quad \text{if } 0 \leq x \leq 4 \text{ and } f(x) \text{ equals zero elsewhere.}$$

Sketch the graph of f(x) and thus i- Evaluate k ii- Find $P(1 \leq X \leq 2)$

$$k = -1/4$$

(c) A pair of fair dice is tossed. Let X assigns to the sum of dices numbers. Calculate the mean, variance and standard deviation of X.

(d) Let X be a random variable with the binomial distribution $b(k;n,p)$.

Prove that $E(X) = np$.

(18 marks)

Question No. 3

(a) A fair die is tossed. Let X denotes twice the number appearing, and let Y denote 1 or 4 according as an odd or an even number appears. Find the probability, expectation, variance and standard deviation of:

i- X ii- Y iii- X+Y iv- XY

iv)

$$p(x) = \begin{cases} \frac{2}{25}x & 0 \leq x \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

3) Prove for any random variable x

i) $E(ax+b) = aE(x) + b$

ii) $V(ax+b) = a^2V(x)$

iii) $E(c) = c$

iv) $V(c) = 0$

4) The heart association claims that only 10% of adults over 30 can pass the physical fitness test. Suppose that four adults are randomly selected and each is given the fitness test.

- a) Find the probability that ^{none}three of the four adults pass the test
- b) Find the probability that three of the four adults pass the test
- c) Let x represent the number of the four adults who pass the test
- d) Drive a formula for p(x), the probability distribution of the binomial random variable x.

5) Refer to problem 4. Use the formula for a binomial random variable to find the probability distribution of x, where x is the number of adults who pass the fitness test, graph the distribution

x	0	1	2	3	4
P(x)	0.6561	0.2916	0.0486	0.0036	0.0001

6) Refer to problem 5. Calculate the mean and the standard deviation.

7) Give a formula for p(x) for a binomial random variable with n=7 and p=0.2

8) Consider the following binomial probability distribution

$$P(x) = \binom{5}{x} (0.7)^x (0.3)^{5-x}, X = 0, 1, 2, 3, 4, 5$$

a) How many trials n are in the experiment?

b) What is the value of p .the probability of success?

c) Graph $p(x)$

d) Find the mean and the standard deviation of x .

9) Suppose X is a binomial random variable with $n = 3$ and $p = 0.3$

a) Calculate the value of $p(x)$, $x=0, 1, 2, 3$, using the formula for a binomial probability distribution.

b) Find the mean and the standard deviation of x

10) If x is a binomial random variable. Calculate mean, variance and standard deviation for each of the following

a) $n = 80$, $p = 0.2$

b) $n = 70$, $p = 0.9$

c) $n = 1000$, $p = 0.04$